THE UNIVERSITY OF TULSA
THE GRADUATE SCHOOL

(FINITE ELEMENT) ANALYSIS OF TORSIONAL SHEAR STRESS, TEMPERATURE PROFILE ALONG THE RADIAL DIRECTION, RADIAL AND LONGITUDINAL DEFLECTION ON ROTARY CEMENT KILN'S SHELL USING ANSYS VALIDATED WITH FORTRAN

Submitted by:
Gembong Baskoro

A project report submitted in partial fulfillment of
the requirements for the degree of Master of Engineering
in the Discipline of Mechanical Engineering

Department of Mechanical Engineering

FALL 94
December 16, 1994
Tulsa, Oklahoma

THE UNIVERSITY OF TULSA
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DEPARTMENT OF MECHANICAL ENGINEERING

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Approved for the discipline of
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By Project Advisor:

James R. Sorem, Jr., Ph.D., P.E.

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Master Project & Report

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ABSTRACT

Rotary Cement Kiln (RKC) has a very important role in the cement manufacturing process to convert raw materials into clinker. Clinker is a solid material in the form of granules with a diameter of 3-25 mm. RCK is in the form of a cylindrical tube that rotates horizontally at a certain tilt angle. Raw materials are burned in the RCK through several stages at different temperatures up to 1500^{o} C. The size of the RCK depends on the capacity of the cement industry. For example, the cement industry with a capacity of 7800 tones/hour, the RCK has dimensions of 84 m in length, 5.6 m in diameter and a rotational speed of 2.8 rpm with an RCK tilt of 4^{o}.

Therefore, RCK must have the strength to be able to withstand operating conditions with high loads and temperatures. This master project aims to analyze the RCK's shell, especially for shear stress, temperature profile, and deflection that occurs in the RCK's shell. This analysis uses two approaches, namely Finite Element Analysis using ANSYS software and the results are validated based on formulas derived from theory and calculated using the FORTRAN program.

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GENERAL INTRODUCTION

In cement industry, Rotary Cement Kiln (RCK) can be divided into two processes: dry, and wet process. The differences of these processes can be seen on the feed materials, also the size of RCK. In wet process the feed materials have the slurry form, and the size of RCK is large enough. On the contrary in dry process the feed materials are dry, and the size of RCK is small. Due to these reasons, most of the cement industries use the dry process.

RCK is used to produce clinkers, as a main material for performing cement, by burning the feed materials. The processes in the RCK can be divided into pre-calcination, calcination, and pre-cooling. These processes related with the chemical reactions. To perform clinker heat generated in the axial direction of RCK, while RCK moving. The Geometry of RCK is a cylinder shell with refractory inside. This construction is supported by three or more pairs of rollers, it depends on the size of RCK. These rollers keep RCK moving freely. The general construction of RCK is shown below.

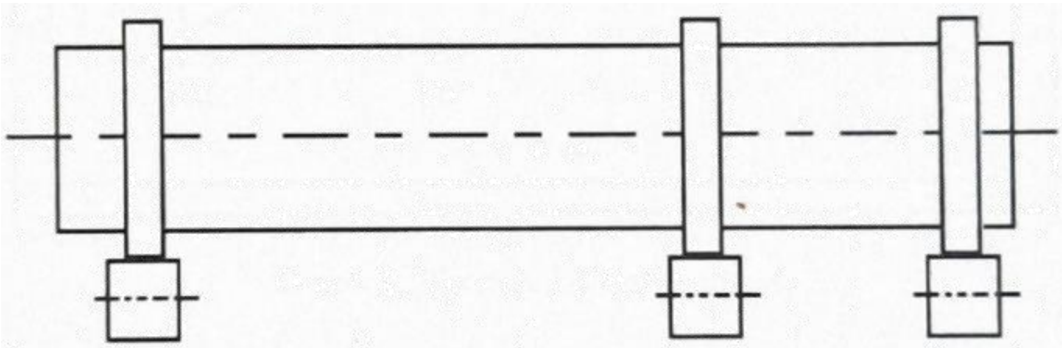


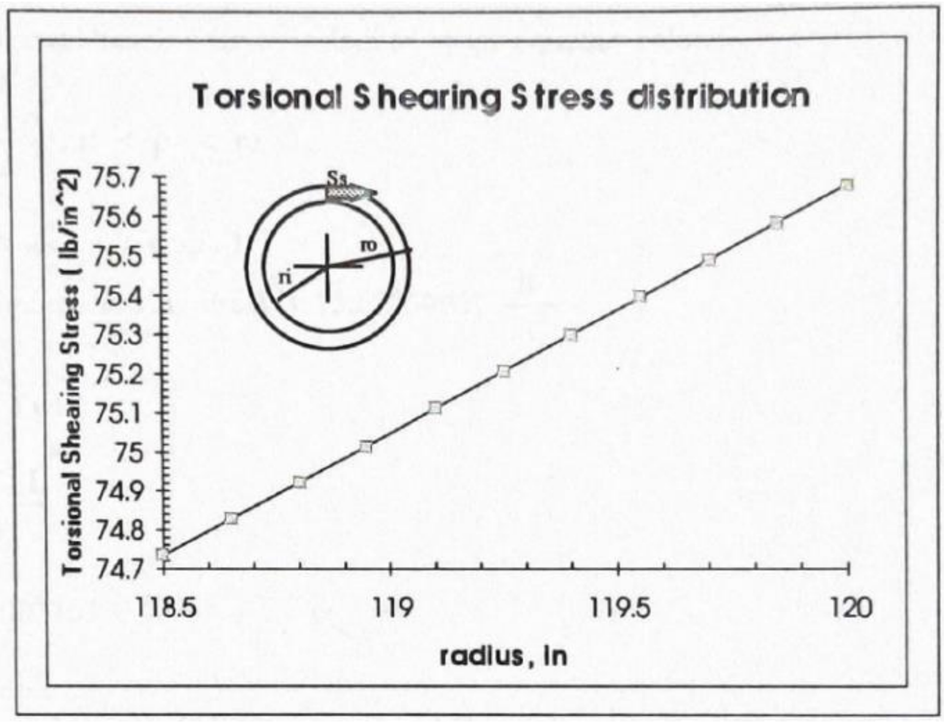
Figure 1. General Construction of a Kiln

The objective of this report is to present the analysis of the kiln's shell. This project is divided into several tasks i.e. ***analysis of the torsional shear stress, temperature profile analysis along radial direction, radial and longitudinal deflection analysis of the kiln's shell***. All the analysis is solved by two methods, theoretical method, and finite element method. The data result is obtained by making FORTRAN programs to support all the information needed.

Analysis of these problems is performed by employing the Finite Element Method and to be compared with the theoretical solution. A software called ANSYS is used to obtain the finite element solution while FORTRAN program is written for theoretical solution.

ANALYSIS OF THE TORSIONAL SHEARING STRESS

The torsional shearing stress along cross-sectional area of Rotary Kiln is defined by the calculation below:



Graph 1. Torsional Shearing Stress

Data Kiln :

L = 1250 in , Length from driven motor to one of the supports
do = 240 in , Outside diameter of kiln
di = 237 in , Inside diameter of shell kiln
HP = 40 HP , Power of electric motor transferred to kiln
n = 0.5 rpm , Speed of kiln
G =1.20 x10^7 $\frac{\text{lb}}{\text{in}^2}$, Modulus of elasticity

Polar moment of inertia :

$J = \frac{\pi}{32}\ (Do^4 - Di^4)$

$J = 7991595.124\ in^4$

Torsion :

$T = \frac{63000 \text{ x HP}}{n}$

T = 5.04 x 10^6 lb.in

The **Torsional shearing stress** is defined by the equation below:

$Ss = \frac{T.\rho}{J}$, ri < ρ < ro

The result shown in graph 1

The maximum shearing stress is 75.67950961 $\frac{lb}{in^2}$

Angle of Twist :

$\theta = \frac{T.L}{G.J}$

$\theta = 0.00376399^o$

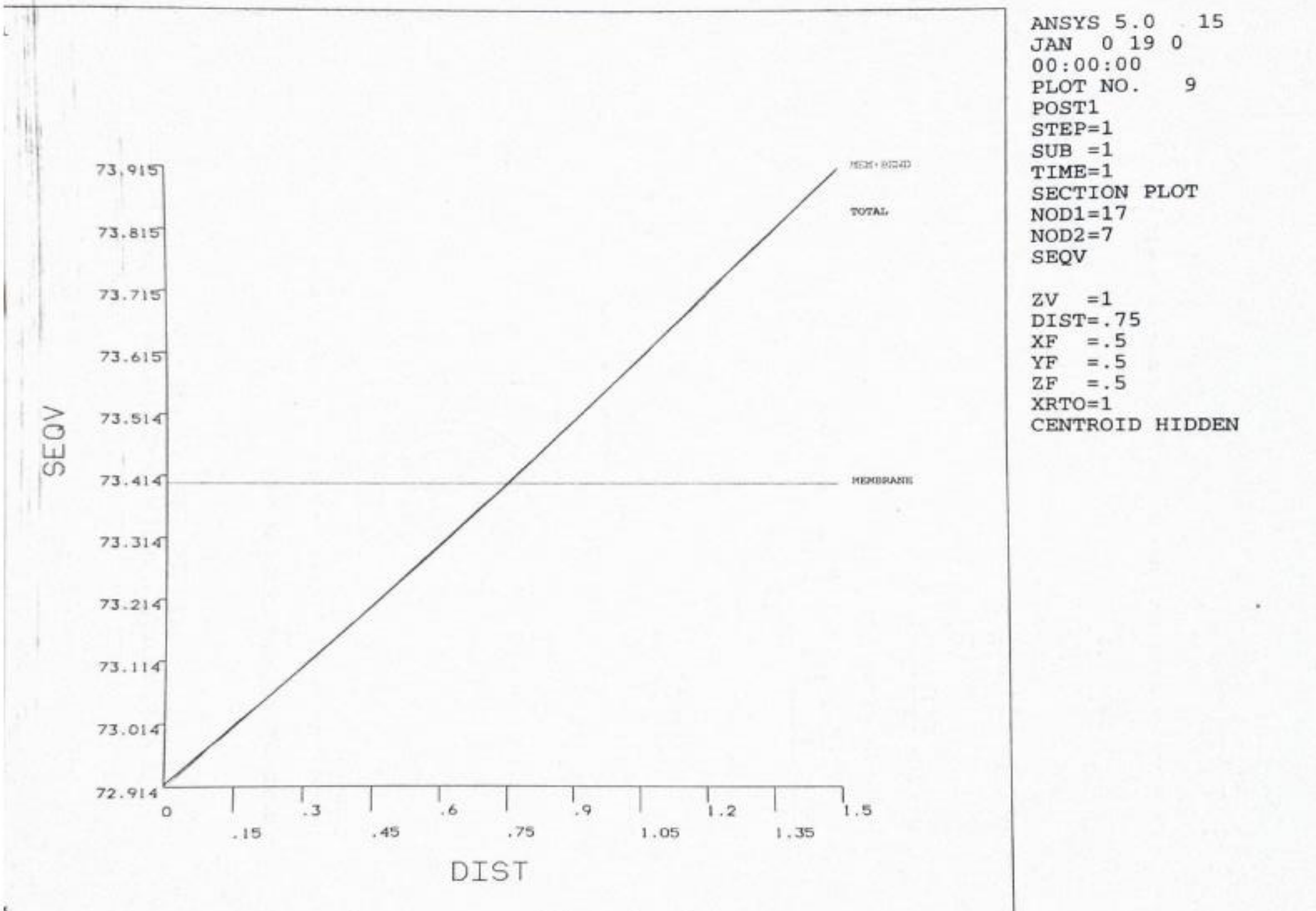
Discussion :

Comparison between ANSYS solution and Calculation:

	Calculation result	**ANSYS Solution**	**Error (%)**
Minimum Shearing Stress	74.733515	72.918	2.429
Maximum Shearing Stress	75.679506	73.913	2334

From this table, the error is small enough. The difference between calculation result and ANSYS solution can be explained as follows: In the actual condition, torsion is transmitted by the electric motor through the gear to the kiln. So that the force acting on the gear is the factor that determines the calculation of the shearing stress. However, the calculation is done by considering the torsion. This is different from the ANSYS solution that considering the force.

In conclusion, the error is caused by the fact that ANSYS consider the force and the calculation does not. However, the error is small enough, so that it is acceptable.



Graph of eqv. stresses along cross section of model

TEMPERATURE PROFILE ANALYSIS ALONG RADIAL DIRECTION OF A KILN

Temperature distribution along r-direction:

The method of conduction on multi layers shell is used to analyze the temperature distribution.

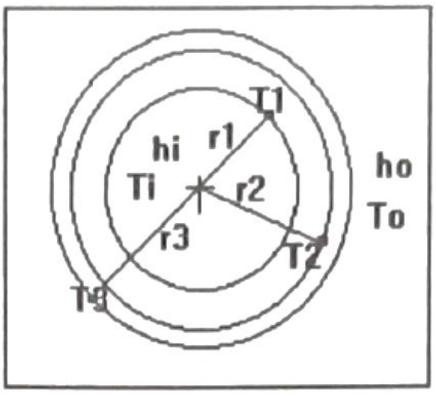


Figure 2. Temperature along r-direction

where :

layer 1 is refractory
layer 2 is shell

Heat per unit length is :

$$\frac{q}{2.\pi.L} = \frac{Ti - To}{\frac{1}{r1.hi} + \sum_{i=1}^{3} . \frac{1}{ki} .\ln\left(\frac{ri+1}{ri}\right) + \frac{1}{r4.ho}}$$

Temperature on each layers are:

$$T1 = Ti - \frac{q}{2.\pi.L} . \frac{1}{r1.hi}$$

$$T2 = T1 - \frac{q}{2.\pi.L} . \frac{\ln\left(\frac{r2}{r1}\right)}{k1}$$

$$T3 = T2 - \frac{q}{2.\pi.L} . \frac{\ln\left(\frac{r3}{r2}\right)}{k2}$$

Temperature distribution along r-direction is solved by the governing equation below:

$$\frac{1}{r} . \frac{d}{dr}\left(r.\frac{dT}{dr}\right) = 0$$

The general equation of temperature distribution is:

T = c1 . ln(r) + c2

where c1 and c2 are constants, and depend on the boundary conditions

at: **ri < r < r1**

Boundary conditions are:
r=ri → T=Ti
r=rl → T=T1
then,

$$c1=\frac{Ti- T1}{\ln\left(\frac{r1}{ri}\right)}$$

c2 = T1 – c1.ln(ri)

at: **r1 < r < r2**

Boundary conditions are:
r=r1 → T=T1
r=r2 → T=T2
then,

$$c1=\frac{T1- T2}{\ln\left(\frac{r1}{r2}\right)}$$

c2 = T1 – c1.ln(r1)

at: **r2 < r < r3**

Boundary conditions are:
r=r2 → T=T2
r=r3 → T=T3
then,

$$c1=\frac{T2- T3}{\ln\left(\frac{r2}{r3}\right)}$$

c2 = T1 – c1.ln(r2)

at: **r3 < r < ro**

Boundary conditions are:
r=r3 → T=T3
r=ro → T=To
then,

$$c1=\frac{To- T3}{\ln\left(\frac{ro}{r3}\right)}$$

c2 = T1 – c1.ln(r3)

All the equations above solved by FORTRAN program (enclosed) with data as follows:

rl =120-1.5-12	(in)	k1=0.0156	Ti=1600 F
r2 =120-1.5	(in)	k2 =0.7833	To=50 F
r3 =120	(in)		hi = 0.0139
			ho= 0.0347

also assume that;

ri =105 (in) at r equal - infinite
ro=130 (in) at r equal + infinite

Result and discussions

Print out result can be seen below:

Temperature on the layers

r(1)=106.5 T(1) = 1463.82300
r(2)=118.5 T(2) = 86.32988
r(3)=120.0 T(3)= 83.09261

r	Temperature
105.0	1600.00200
106.0	1509.00300
107.0	1403.39400
108.0	1283.37700
109.0	1164.46600
110.0	1046.64100
111.0	929.88210
112.0	814.17070
113.0	699.48780
114.0	585.81530
115.0	473.13560
116.0	361.43150
117.0	250.68620
118.0	140.88350
119.0	85.24620
120.0	83.09255
121.0	79.66168
1220	76.25889
123.0	72.88387
1240	69.53619
125.0	66.21539
126.0	62.92106

127.0	59.65276
128.0	56.41010
129.0	53.19268
130.0	50.00010

Discussions :

The comparison between theoretical data and ANSYS solution appear on the table below:

	Theoretical	ANSYS	Error (%)
Max. Temperature	1463.823	1465	0.08
Min. Temperature	83.09261	97.952	15.17

Temperature profile is shown in the graph below.

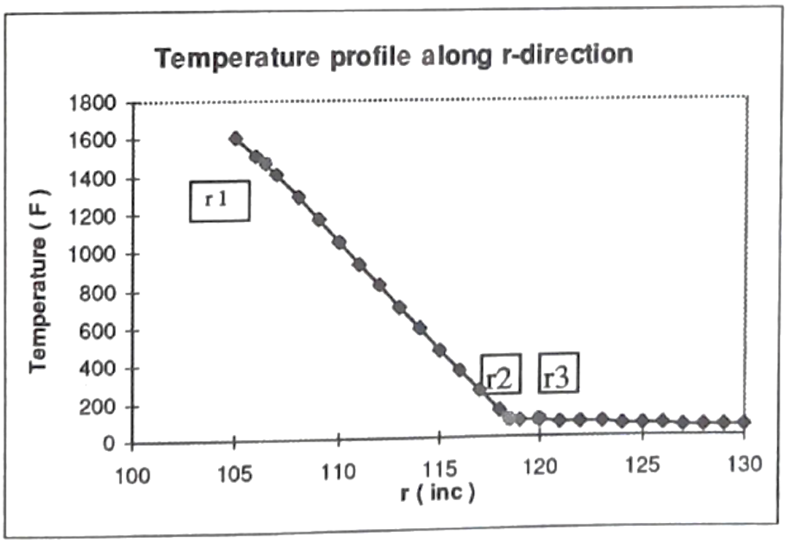
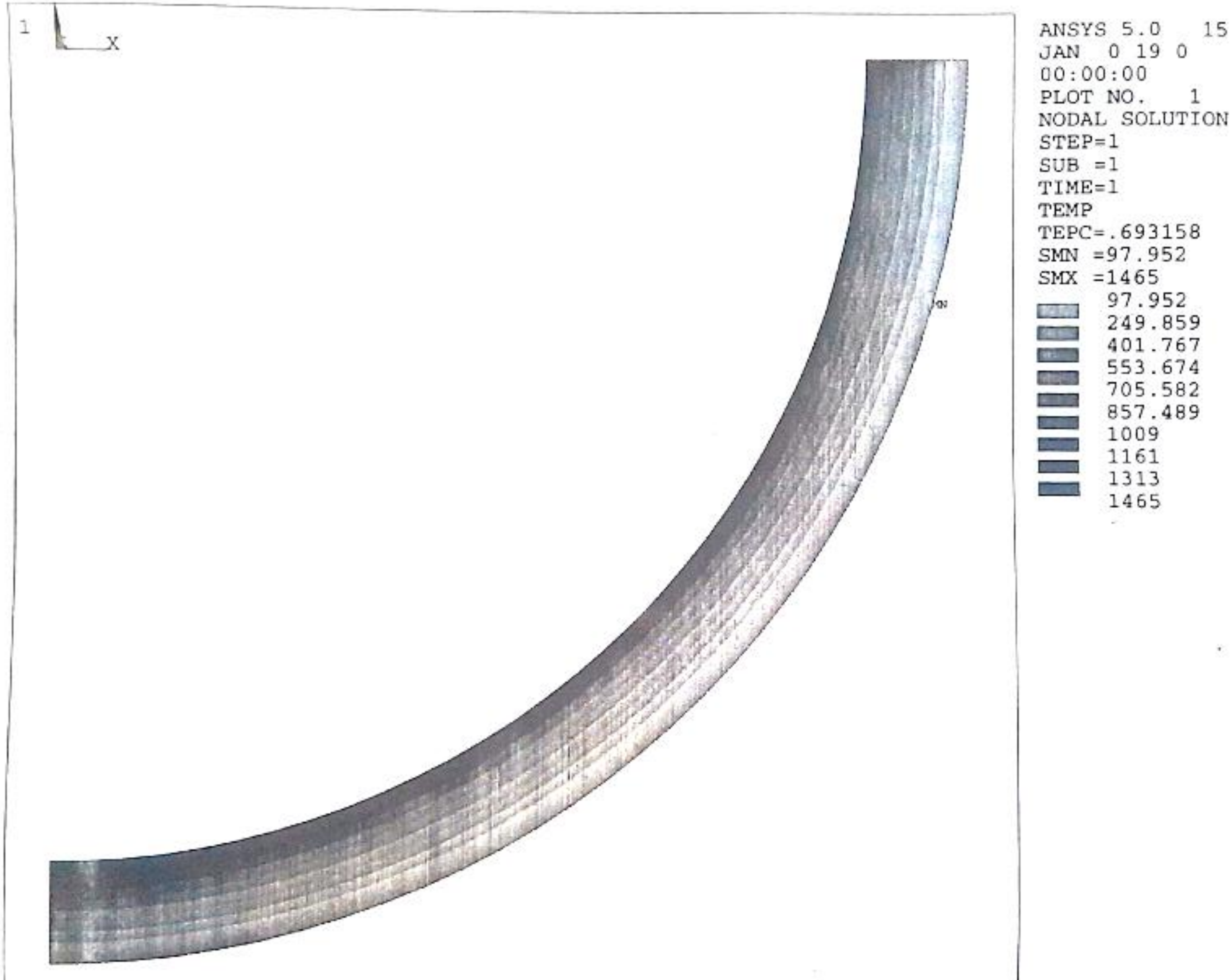


Figure 3. Temperature profile

The graph shows that between r1 and r2 temperature drops very rapidly, also it looks linear. These conditions occur because the material, Magnesite Chrome brick, behaves as insulating material, and the linearity of the graph caused by the slope between every point is very small. The comparison between ANSYS and theoretical solution shows that they are very close for predicting max. and min. temperature. It means that ANSYS solution is capable of predicting temperature profile in the Kiln.



Temperatur distribution on radial direction of a kiln

RADIAL DEFLECTION ANALYSIS OF THE KILN'S SHELL

The deformation of kiln's shell can be predicted by making a model with the assumption that long is uniformly distributed in the specific area and neglects the rotation of kiln. This assumption can be described as seen in figure 1. The load is labelled a5 Q/2, and the reaction as R.

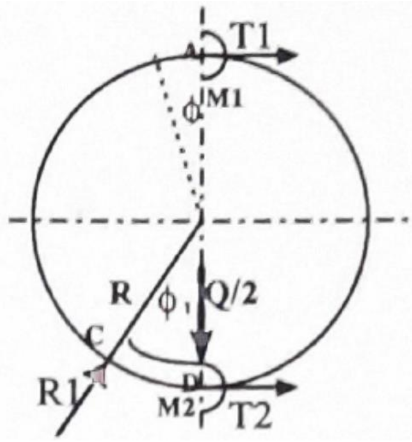


Figure 4. The model of kiln's shell

Reaction on the support:

$$R1 = R2 = \frac{Q}{2\mathrm{Cos}\phi 1}$$

Moments :

$$M_A^C = \frac{QR}{2\pi}\ (\mathrm{Sec}\phi 1 - 1 - \phi 1.\tan\phi 1.\mathrm{Cos}\phi)$$

$$M_C^D = \frac{QR}{2\pi}\ (\mathrm{Sec}\phi 1 - 1 - \phi 1.\tan\phi 1.\mathrm{Cos}\phi) + \frac{QR}{2\mathrm{Cos}\phi 1}\ \mathrm{Sin}\ (\phi + \phi 1)$$

in dimensionless :

$$\frac{M_C^A}{QR} = \frac{1}{2\pi}\ (\mathrm{Sec}\phi 1 - 1 - \phi 1.\tan\phi 1.\mathrm{Cos}\phi)$$

$$\frac{M_D^C}{QR} = \frac{1}{2\pi}\ (\mathrm{Sec}\phi 1 - 1 - \phi 1.\tan\phi 1.\mathrm{Cos}\phi) + \frac{1}{2\mathrm{Cos}\phi 1}\ \mathrm{Sin}\ (\phi + \phi 1)$$

The general expression of deflection (Δ), in terms of elastic energy:

$$\Delta = \int \frac{m.M.ds}{F.E.I}$$

where:

m = Bending moment caused by auxiliary force F
M = Bending moment caused by actual load Q
F = Represent an auxiliary force

so that:

$m = F.R.\ Sin\phi$
$ds = R.d\phi$
$I = \frac{b.t^3}{12}$

where: t = thickness of the shell

Then the equation of radial deflection becomes:

$$\Delta_D^C = \frac{1}{E.I} \int_D^C R^2.\text{Sin}\,\phi.\,M_D^C\ d\phi$$

$$\Delta_C^A = \frac{1}{E.I} \int_C^A R^2.\text{Sin}\,\phi.\,M_C^A\ d\phi$$

Stress :

$$\tau = \frac{M.c}{I}$$

where:

$$c = \frac{t}{2}$$

Then after substituting the momentum equation into stress equation, it is given:

$$\tau\,{}_D^C = \frac{6}{b.t^2}.\ M_C^D$$

Also,

$$\tau\,{}_C^A = \frac{6}{b.t^2}.\ M_C^A$$

From these equations the moment distribution can usually be related to stress distribution and displacement. On the other hand, moment distribution can describe all the conditions happening on the kiln's shell. The moment distribution can be seen in the graph below.

The data for generating this graph is taken from the data of the kiln as seen in the previous analysis.

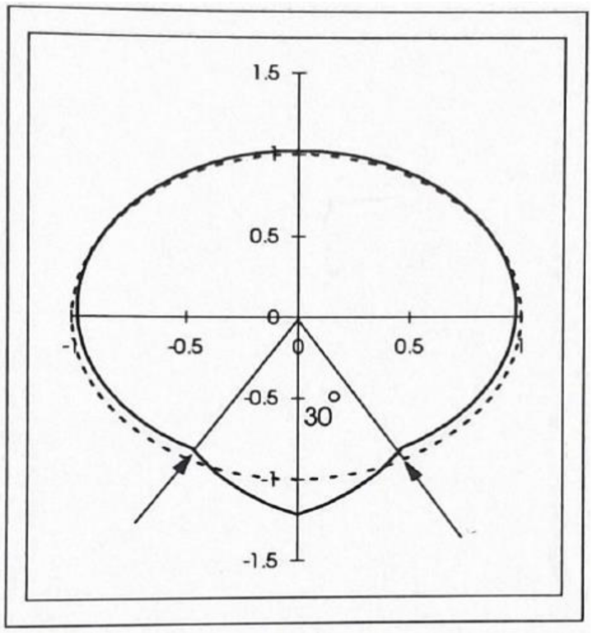
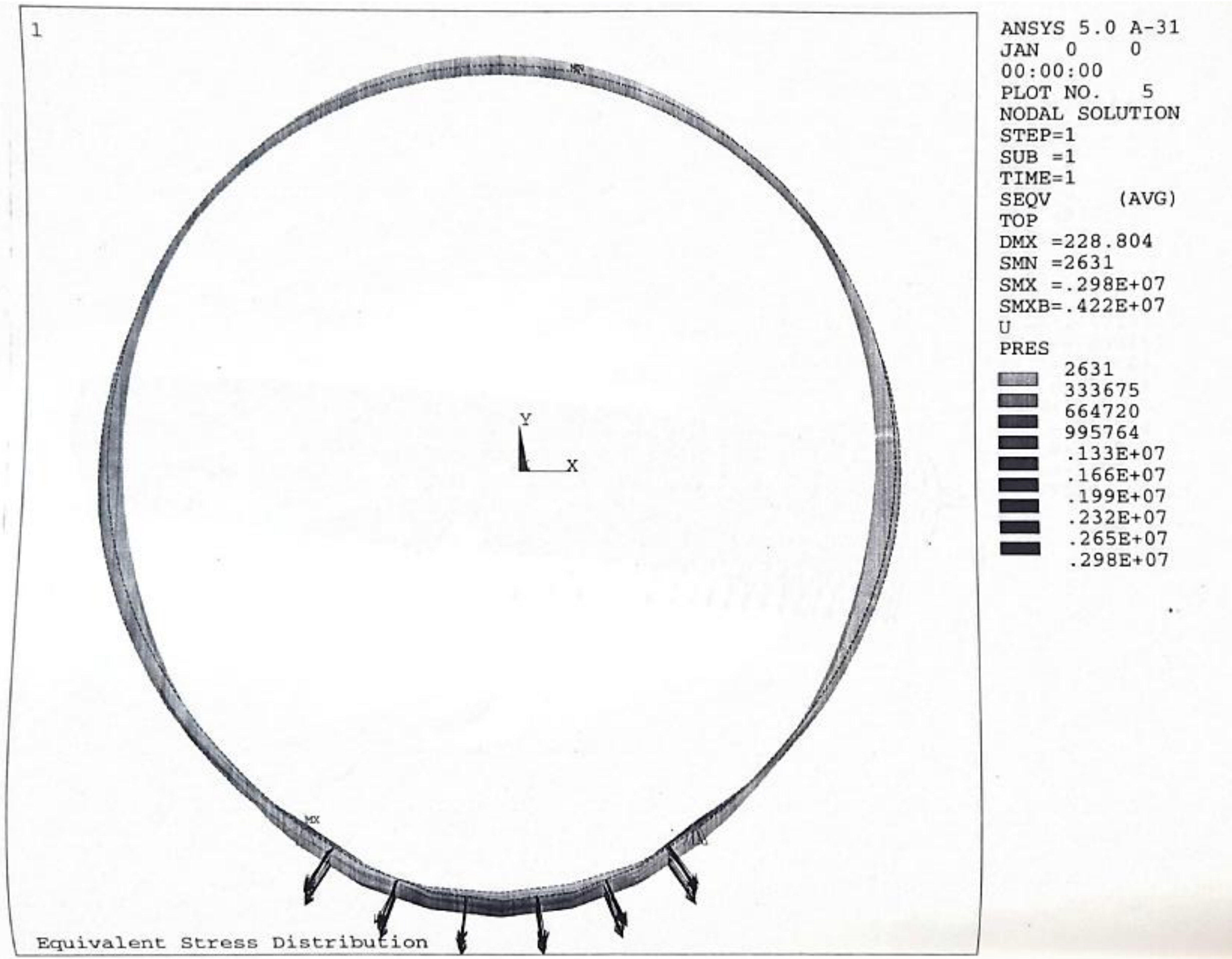


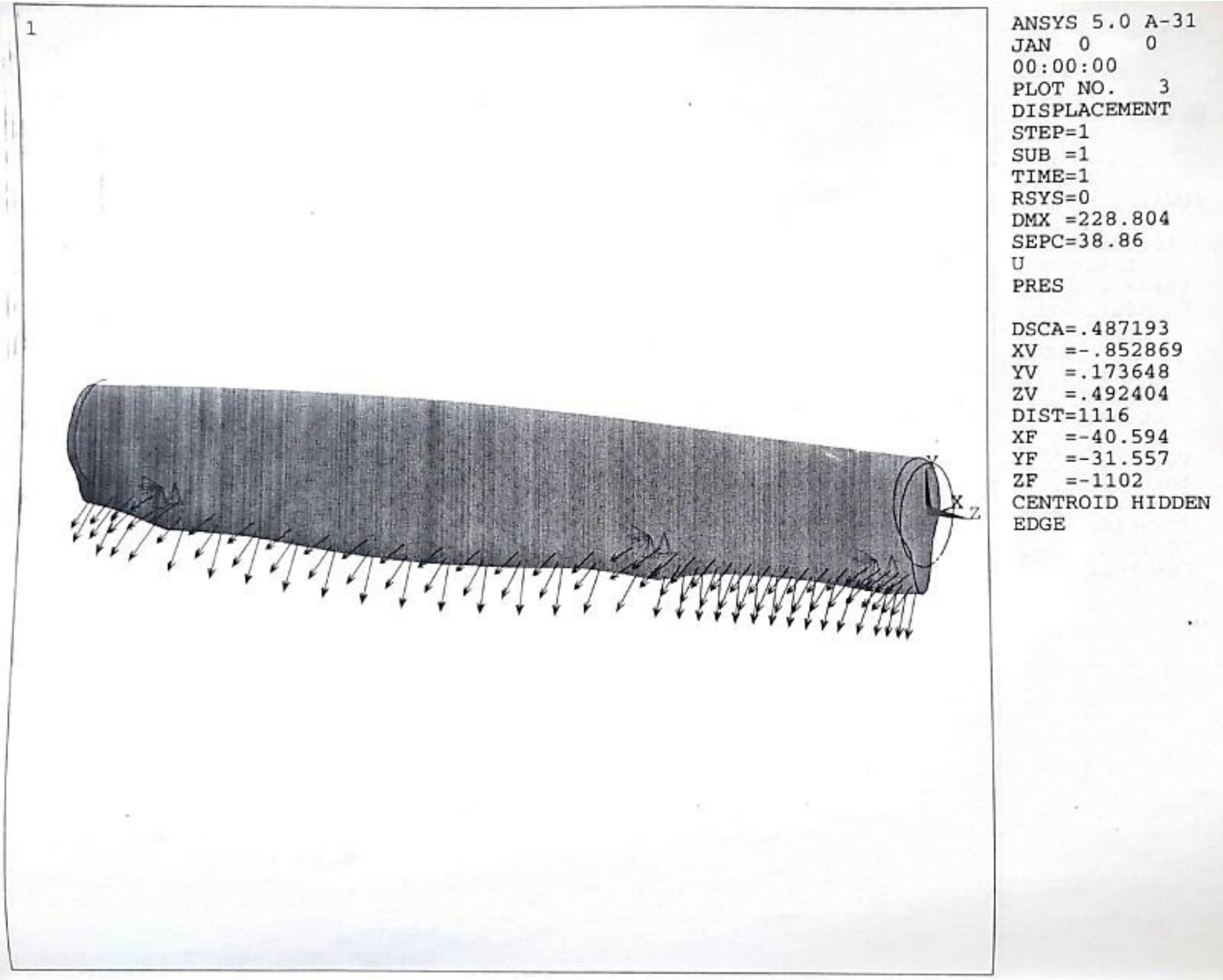
Figure 5. Moment distribution

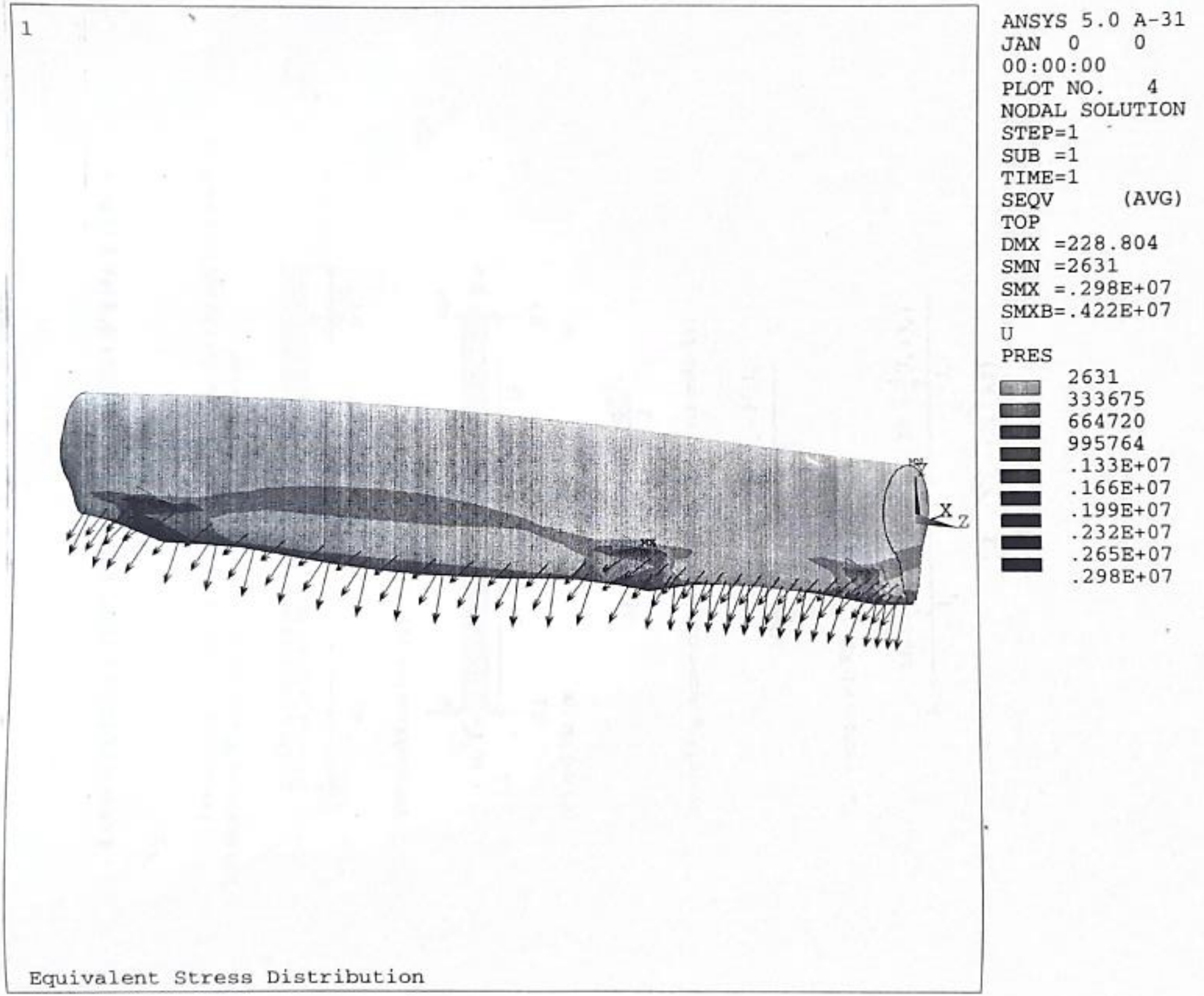
The trend of these results agree with the ANSYS solution.

Conclusions:

Due to the load distribution, shell tends to deform in the radial direction. It also deforms the shell in the longitudinal direction. The maximum deflection occurs at the long distance of the supports, or there is similar to cantilever beam. However, in the radial direction there are very small deflection. From these results, it is reasonable to give more attention to the deflection in the longitudinal direction instead of radial direction.


1
ANSYS 5.0 A-31
JAN 0 0
00:00:00
PLOT NO. 5
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SEQV (AVG)
TOP
DMX =228.804
SMN =2631
SMX =.298E+07
SMXB=.422E+07
U
PRES
2631
333675
664720
995764
.133E+07
.166E+07
.199E+07
.232E+07
.265E+07
.298E+07
Y
X
MX
Equivalent Stress Distribution

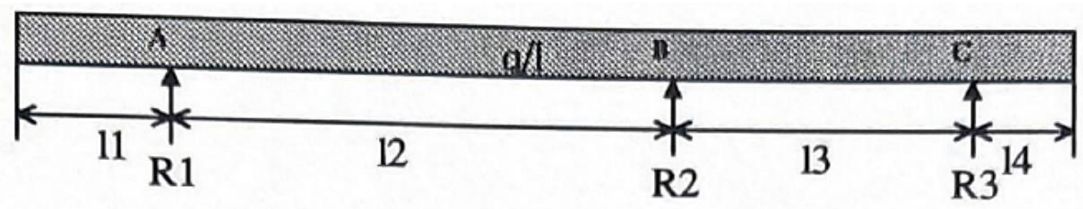

1
ANSYS 5.0 A-31
JAN 0 0
00:00:00
PLOT NO. 3
DISPLACEMENT
STEP=1
SUB =1
TIME=1
RSYS=0
DMX =228.804
SEPC=38.86
U
PRES
DSCA=.487193
XV =-.852869
YV =.173648
ZV =.492404
DIST=1116
XF =-40.594
YF =-31.557
ZF =-1102
CENTROID HIDDEN
EDGE
Y
X
Z



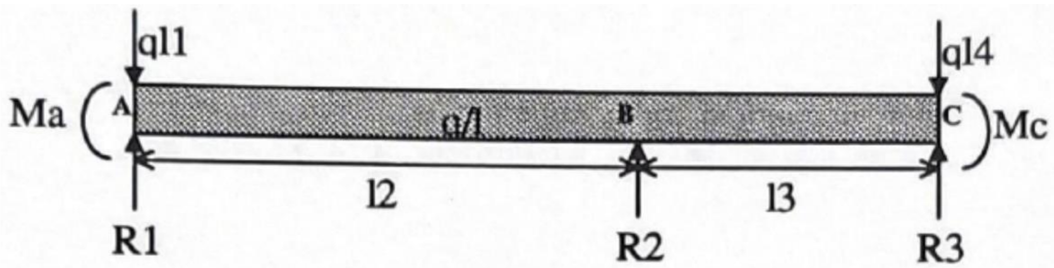
Equivalent Stress Distribution

LONGITUDINAL DEFLECTION ANALYSIS OF THE KILN'S SHELL

Longitudinal deflection of the kiln can also be determined by making a model of kiln as a beam. The kiln has three supports on it as shown below:



This model can be simplified as



Ma and Mc are

$$Ma = \frac{-ql_1^2}{2}, \; Mc = \frac{-ql_4^2}{2}$$

By applying three-moment method, Mb can be determined as

$$Mb = \frac{-\frac{q}{4}\left(l_2^3 + l_3^3\right) - Ma.l_2 - Mc.l_3}{2(l_2 - l_3)}$$

The reactions R1, R2 and R3 are

$$R1 = \frac{Mb + \frac{q}{2}(l_1 - l_2)^2}{l_2}, \; R3 = \frac{Mb + \frac{q}{2}(l_3 - l_4)^2}{l_3}$$

$$R2 = q.\sum_{i=1}^{4} li - (R1 + R2)$$

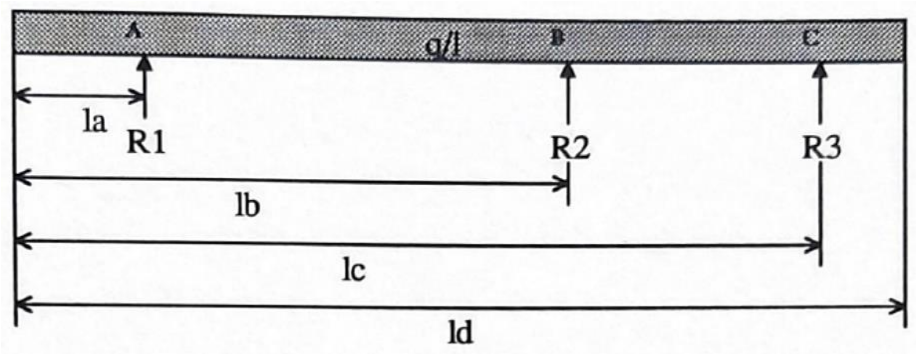
Shearing force

The shearing force in each section are.

$0<x<l_1$ $v(x) = -qx$
$l_1 <x< l_2$ $v(x) = -qx +R1$
$l_2 <x< l_3$ $v(x) = -qx + R1+R2$
$l_3 <x< l_4$ $v(x) = -qx +R1+R2+R3$

Bending Moment

The moment in each section can be derived based on the beam shown below:



Where

la=l1
lb=l1+l2
lc=l1+l2+l3
ld=l1+l2+l3+l4

The moment distribution are

$0 < x < l_a$ $M(x) = -\frac{qx}{2}$

$l_a < x < l_b$ $M(x) = -\frac{qx}{2} +R1(x - l_a)$

$l_b < x < l_c$ $M(x) = -\frac{qx}{2} +R1(x - l_a) + R2(x - l_b)$

$l_c < x < l_d$ $M(x) = -\frac{qx}{2} +R1(x - l_a) + R2(x - l_b) + R3(x - l_c)$

Deflection:

The deflection can be derived by the equations below:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{E.I}$$

Boundary conditions for this problem are:

$$y(la)\Big|_{0}^{l_a} = y(la)\Big|_{l_a}^{l_b} = 0, \qquad @la \rightarrow \qquad \frac{dy}{dx}\Big|_{0}^{l_a} = \frac{dy}{dx}\Big|_{l_a}^{l_b}$$

$$y(lb)\Big|_{l_a}^{l_b} = y(lb)\Big|_{l_b}^{l_c} = 0, \qquad @lb \rightarrow \qquad \frac{dy}{dx}\Big|_{l_a}^{l_b} = \frac{dy}{dx}\Big|_{l_b}^{l_c}$$

$$y(lc)\Big|_{l_b}^{l_c} = y(lc)\Big|_{l_c}^{l_d} = 0, \qquad @lc \rightarrow \qquad \frac{dy}{dx}\Big|_{l_b}^{l_c} = \frac{dy}{dx}\Big|_{l_c}^{l_d}$$

The deflections are:

$0<x<l_a$, $\quad y(x) = -\frac{1}{E.I}\left[-\frac{qx^4}{24} + C_1x + C_2\right]$

$l_a<x<l_b$, $\quad y(x) = -\frac{1}{E.I}\left[-\frac{qx^4}{24} + \frac{R_1x^3}{6} - \frac{R_1l_ax^2}{2} + C_3x + C_4\right]$

$l_b<x<l_c$, $\quad y(x) = -\frac{1}{E.I}\left[-\frac{qx^4}{24} + \frac{R_1x^3}{6} - \frac{R_1l_ax^2}{2} + \frac{R_2x^3}{6} - \frac{R_2l_bx^2}{2} + C_5x + C_6\right]$

$l_c<x<l_d$, $\quad y(x) = -\frac{1}{E.I}\left[-\frac{qx^4}{24} + \frac{R_1x^3}{6} - \frac{R_1l_ax^2}{2} + \frac{R_2x^3}{6} - \frac{R_2l_bx^2}{2} + \frac{R_3x^3}{6} - \frac{R_3l_cx^2}{2} C_7x + C_8\right]$

After applying the boundary conditions into the equations of deflection, the constants can be determined as follows:

$$C_3 = \frac{1}{(l_b - l_a)}\left[\frac{q}{24}(l_b^4 - l_a^4) + \frac{R_1}{2}\left(\frac{2l_a^3}{3} - \frac{l_b^3}{3} + l_a\, l_b^2\right)\right]$$

$$C_4 = \frac{ql_a^4}{24} + \frac{R_1l_a^3}{3} - C_3l_a$$

$$C_1 = C_3 - \frac{R_1l_a^2}{2}$$

$$C_2 = \frac{ql_a^4}{24} - C_1\, l_a$$

$$C_5 = C_3 + \frac{R_2l_b^2}{2}$$

$$C_6 = \frac{ql_b^4}{24} - \frac{l_b^3}{6}(R_1 + R_1) + \frac{R_1l_a\, l_b^2}{2} + \frac{R_2l_b^3}{2} - C_5\, l_b$$

$$C_7 = C_5 + \frac{R_3l_c^2}{2}$$

$$C_8 = \frac{ql_c^4}{24} - \frac{l_c^3}{6}(R_1 + R_2 + R_1) + \frac{l_c^2}{2}(R_1l_a + R_2l_b + R_3l_c) - C_7\, l_c$$

Calculation example:

Take data input as follows:

q =150 lb/in
l1 =200 in
l2 = 1300 in
l3 =600 in
l4 =100 in

do = 243 in, Outside diameter of shell
di = 237 in, Inside diameter of shell
E = 1.2 E7 1b/in^2 (Modulus of Elasticity)

Results :

Moment of Inertia (I) :

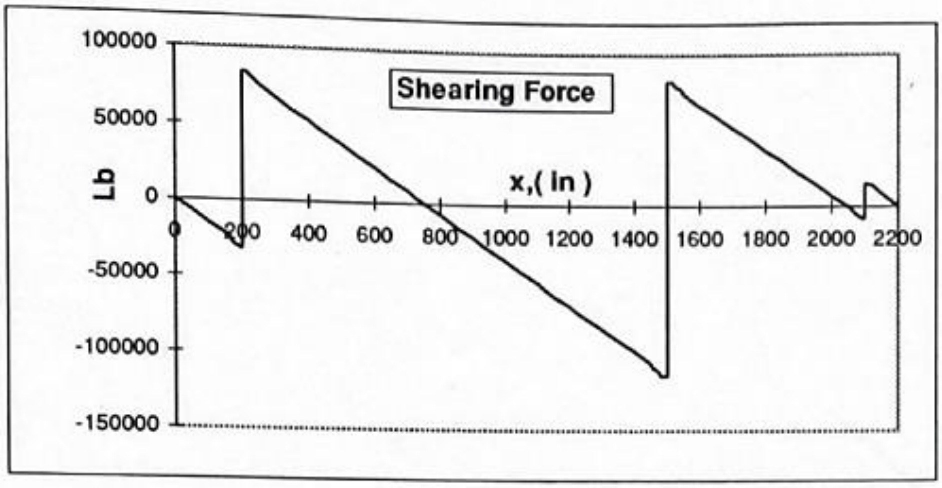
$I = \frac{\pi.(d_o^4 - d_i^4)}{64} = 16288561$ (in^2), Moment Inertia for hollow cylinder

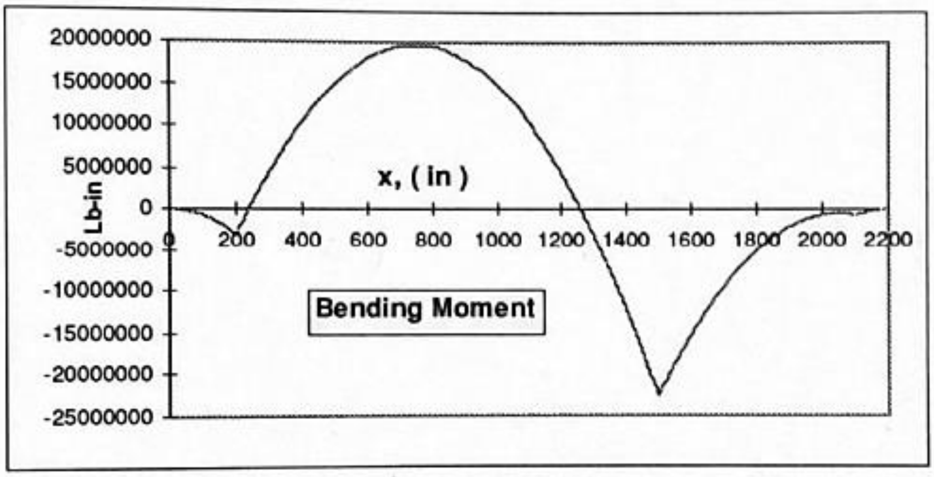
Moment at the support:
Ma = -3000000 lb-in
Mb = -2.37E7 lb-in
Mc = -750000 lb-in

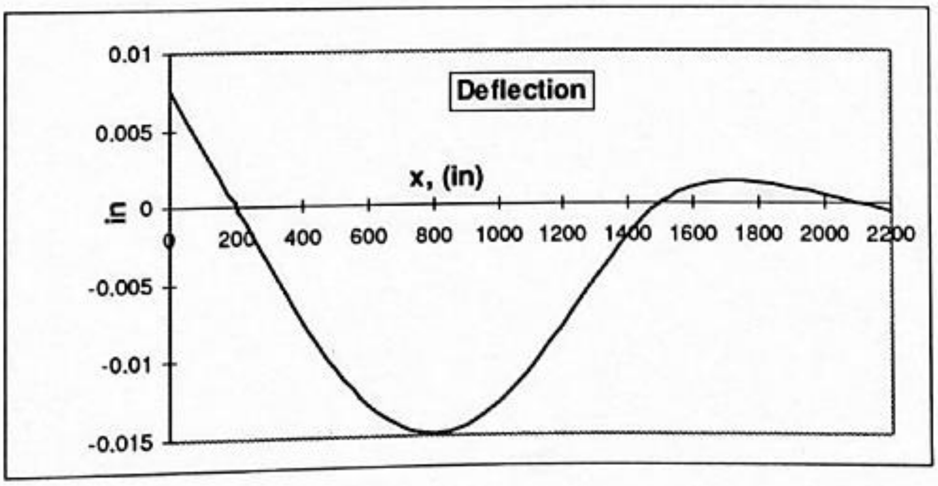
Reactions at the support (R)
R1 = 112371 lb
R2 = 1941587 lb
R3 = 23470.39 lb

Maximum deflection :
ymax = 0.01482 in

Trend of the parameters are as follows :







Conclusion:

The beam model is not capable of analyzing the behavior of the kiln shell. The major reason for the difficulties of the beam model is in the support. Kiln shell in practice is supported by two rollers in each section so that it is different with beam model that consider one support in each section. Also, in practice the load is distributed inside of the kiln shell, however in beam model the load is distributed outside the beam.

In general, the beam model can usually be used to predict the trend of the deflection of kiln shell. This is possible because in the deflection formula there is term I (. moment of inertia), so that it can be used for different shapes of model, such as hollow cylinder.

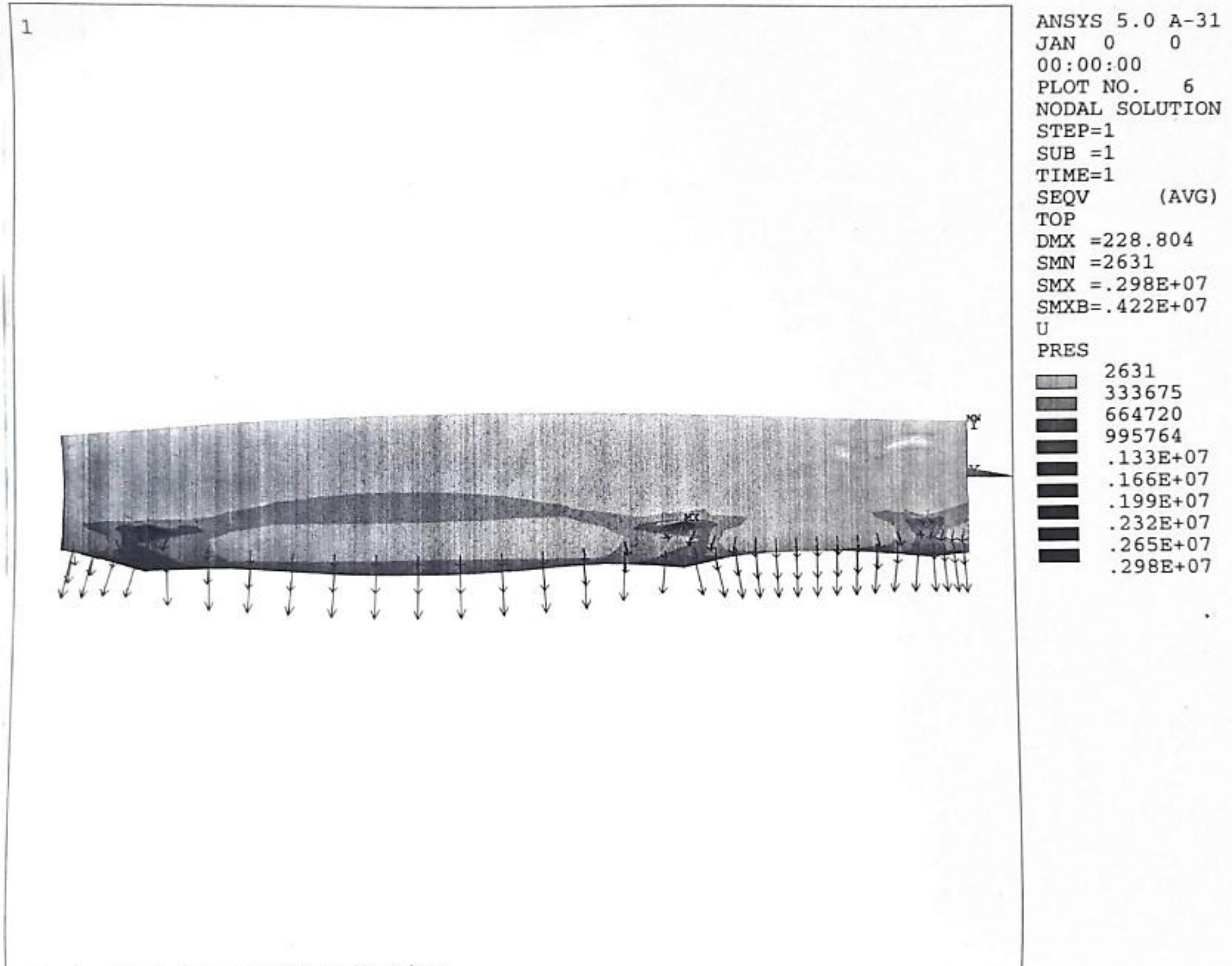
From the trend line of the deflection, maximum deflection occurs in the position where two supports have the longest distance. On the other hand, it happens at the position where maximum bending moment occurs.

Ma = -.30000E+07 Mb = -.22668E+08 Mc = -.75000E+06
R1 = 0.11237E+06 R2 = 0.19416E+06 R3 = 0.23470E+05

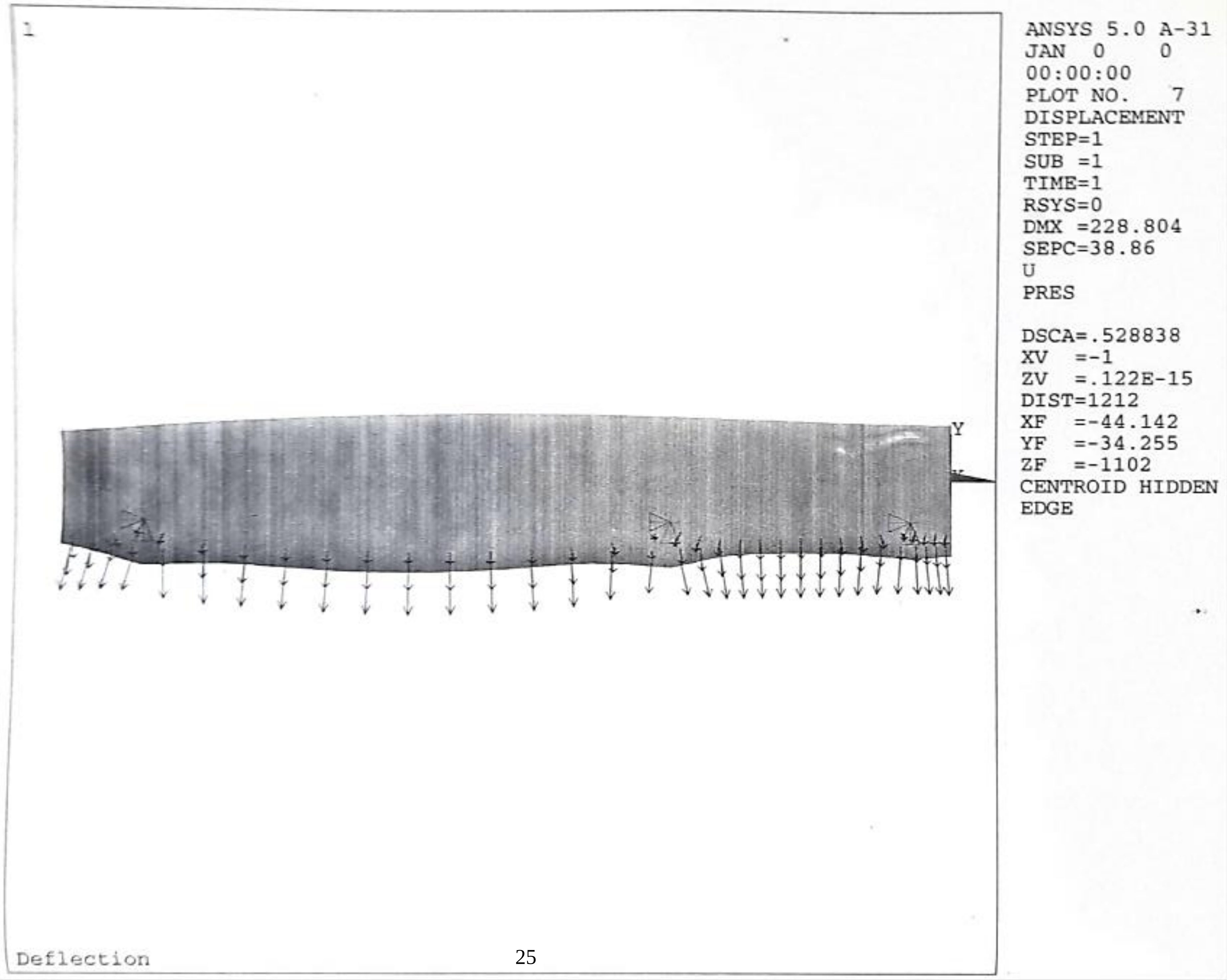
x (in)	Shearing force (lb)	Bending Moment (lb-in)	Deflection (in)
0.0	0.00000E+00	0.00000E+00	0.75410E-02
25.0	-.37500E+04	-.46875E+05	0.66047E-02
50.0	-.75000E+04	-.18750E+06	0.56683E-02
75.0	-.11250E+05	-.42188E+06	0.47313E-02
100.0	-.15000E+05	-.75000E+06	0.37929E-02
125.0	-.18750E+05	-.11719E+07	0.28520E-02
150.0	-.22500E+05	-.16875E+07	0.19074E-02
175.0	-.26250E+05	-.22969E+07	0.95740E-03
200.0	-.30000E+05	-.30000E+07	0.36672E-10
200.0	0.82371E+05	-.30000E+07	-.13097E-09
225.0	0.78621E+05	-.98760E+06	-.96552E-03
250.0	0.74871E+05	0.93105E+06	-.19342E-02
275.0	0.71121E+05	0.27559E+07	-.29000E-02
300.0	0.67371E+05	0.44871E+07	-.38569E-02
325.0	0.63621E+05	0.61245E+07	-.47996E-02
350.0	0.59871E+05	0.76681E+07	-.57227E-02
375.0	0.56121E+05	0.91180E+07	-.66212E-02
400.0	0.52371E+05	0.10474E+08	-.74907E-02
425.0	0.48621E+05	0.11737E+08	-.83267E-02
450.0	0.44871E+05	0.12905E+08	-.91252E-02
475.0	0.41121E+05	0.13980E+08	-.98824E-02
500.0	0.37371E+05	0.14961E+08	-.10595E-01
525.0	0.33621E+05	0.15849E+08	-.11260E-01
550.0	0.29871E+05	0.16642E+08	-.11874E-01
575.0	0.26121E+05	0.17342E+08	-.12435E-01
600.0	0.22371E+05	0.17948E+08	-.12940E-01
625.0	0.18621E+05	0.18461E+08	-.13388E-01
650.0	0.14871E+05	0.18879E+08	-.13777E-01
675.0	0.11121E+05	0.19204E+08	-.14106E-01
700.0	0.73709E+04	0.19435E+08	-.14374E-01
725.0	0.36209E+04	0.19573E+08	-.14579E-01
750.0	-.12905E+03	0.19617E+08	-.14722E-01
775.0	-.38791E+04	0.19566E+08	-.14802E-01
800.0	-.76291E+04	0.19423E+08	-.14819E-01
825.0	-.11379E+05	0.19185E+08	-.14775E-01
850.0	-.15129E+05	0.18854E+08	-.14669E-01
875.0	-.18879E+05	0.18429E+08	-.14502E-01
900.0	-.22629E+05	0.17910E+08	-.14277E-01
925.0	-.26379E+05	0.17297E+08	-.13995E-01
950.0	-.30129E+05	0.16591E+08	-.13657E-01
975.0	-.33879E+05	0.15791E+08	-.13267E-01
1000.0	-.37629E+05	0.14897E+08	-.12826E-01
1025.0	-.41379E+05	0.13909E+08	-.12337E-01
1050.0	-.45129E+05	0.12828E+08	-.11804E-01
1075.0	-.48879E+05	0.11653E+08	-.11230E-01
1100.0	-.52629E+05	0.10384E+08	-.10618E-01
1125.0	-.56379E+05	0.90213E+07	-.99737E-02
1150.0	-.60129E+05	0.75649E+07	-.93003E-02
1175.0	-.63879E+05	0.60148E+07	-.86027E-02
1200.0	-.67629E+05	0.43709E+07	-.78860E-02
1225.0	-.71379E+05	0.26333E+07	-.71553E-02
1250.0	-.75129E+05	0.80199E+06	-.64162E-02
1275.0	-.78879E+05	-.11231E+07	-.56745E-02
1300.0	-.82629E+05	-.31420E+07	-.49365E-02
1325.0	-.86379E+05	-.52546E+07	-.42085E-02
1350.0	-.90129E+05	-.74609E+07	-.34974E-02
1375.0	-.93879E+05	-.97610E+07	-.28102E-02
1400.0	-.97629E+05	-.12155E+08	-.21541E-02
1425.0	-.10138E+06	-.14642E+08	-.15370E-02
1450.0	-.10513E+06	-.17224E+08	-.96676E-03
1475.0	-.10888E+06	-.19899E+08	-.45157E-03
1500.0	-.11263E+06	-.22668E+08	-.26152E-07
1500.0	0.81530E+05	-.22668E+08	0.36479E-06
1525.0	0.77780E+05	-.20676E+08	0.38176E-03
1550.0	0.74030E+05	-.18779E+08	0.69746E-03
1575.0	0.70280E+05	-.16975E+08	0.95270E-03
1600.0	0.66530E+05	-.15265E+08	0.11539E-02
1625.0	0.62780E+05	-.13648E+08	0.13061E-02
1650.0	0.59030E+05	-.12126E+08	0.14148E-02
1675.0	0.55280E+05	-.10697E+08	0.14844E-02
1700.0	0.51530E+05	-.93619E+07	0.15200E-02
1725.0	0.47780E+05	-.81205E+07	0.15256E-02
1750.0	0.44030E+05	-.69729E+07	0.15053E-02

1775.0	0.40280E+05	-.59190E+07	0.14626E-02
1800.0	0.36530E+05	-.49589E+07	0.14008E-02
1825.0	0.32780E+05	-.40925E+07	0.13234E-02
1850.0	0.29030E+05	-.33199E+07	0.12327E-02
1875.0	0.25280E+05	-.26410E+07	0.11314E-02
1900.0	0.21530E+05	-.20559E+07	0.10215E-02
1925.0	0.17780E+05	-.15646E+07	0.90522E-03
1950.0	0.14030E+05	-.11669E+07	0.78413E-03
1975.0	0.10280E+05	-.86309E+06	0.65881E-03
2000.0	0.65296E+04	-.65298E+06	0.53099E-03
2025.0	0.27796E+04	-.53661E+06	0.40061E-03
2050.0	-.97040E+03	-.51397E+06	0.26919E-03
2075.0	-.47204E+04	-.58514E+06	0.13551E-03
2100.0	-.84704E+04	-.75002E+06	0.15021E-06
2100.0	0.15000E+05	-.75002E+06	0.17167E-06
2125.0	0.11250E+05	-.42189E+06	-.13727E-03
2150.0	0.75000E+04	-.18750E+06	-.27608E-03
2175.0	0.37500E+04	-.46864E+05	-.41595E-03
2200.0	-.19531E-02	-.16250E+02	-.55498E-03

Maximum Bending Moment = 0.20676E+08 lb-in
Maximum Deflection = 0.14819E-01 in



Equivalent Stress Distribution


1
ANSYS 5.0 A-31
JAN 0 0
00:00:00
PLOT NO. 7
DISPLACEMENT
STEP=1
SUB =1
TIME=1
RSYS=0
DMX =228.804
SEPC=38.86
U
PRES
DSCA=.528838
XV =-1
ZV =.122E-15
DIST=1212
XF =-44.142
YF =-34.255
ZF =-1102
CENTROID HIDDEN
EDGE
Y
Deflection

REFERENCES

1. **ANSYS** User's Manual Revision 5.0
2. David. S Burnett, **Finite Element Analysis**, Addison-Wesley, 1988
3. Shigley and Mischke, **Engineering Mechanical Design**, McGraw-Hill, 1989
4. Timoshenko and Young, **Theory of Structures**, McGraw-Hill, 1965
5. Van Den Broek, **Elastic Energy Theory**, John Wiley and Sons, 1942

APPENDIX

ANSYS SOLUTION

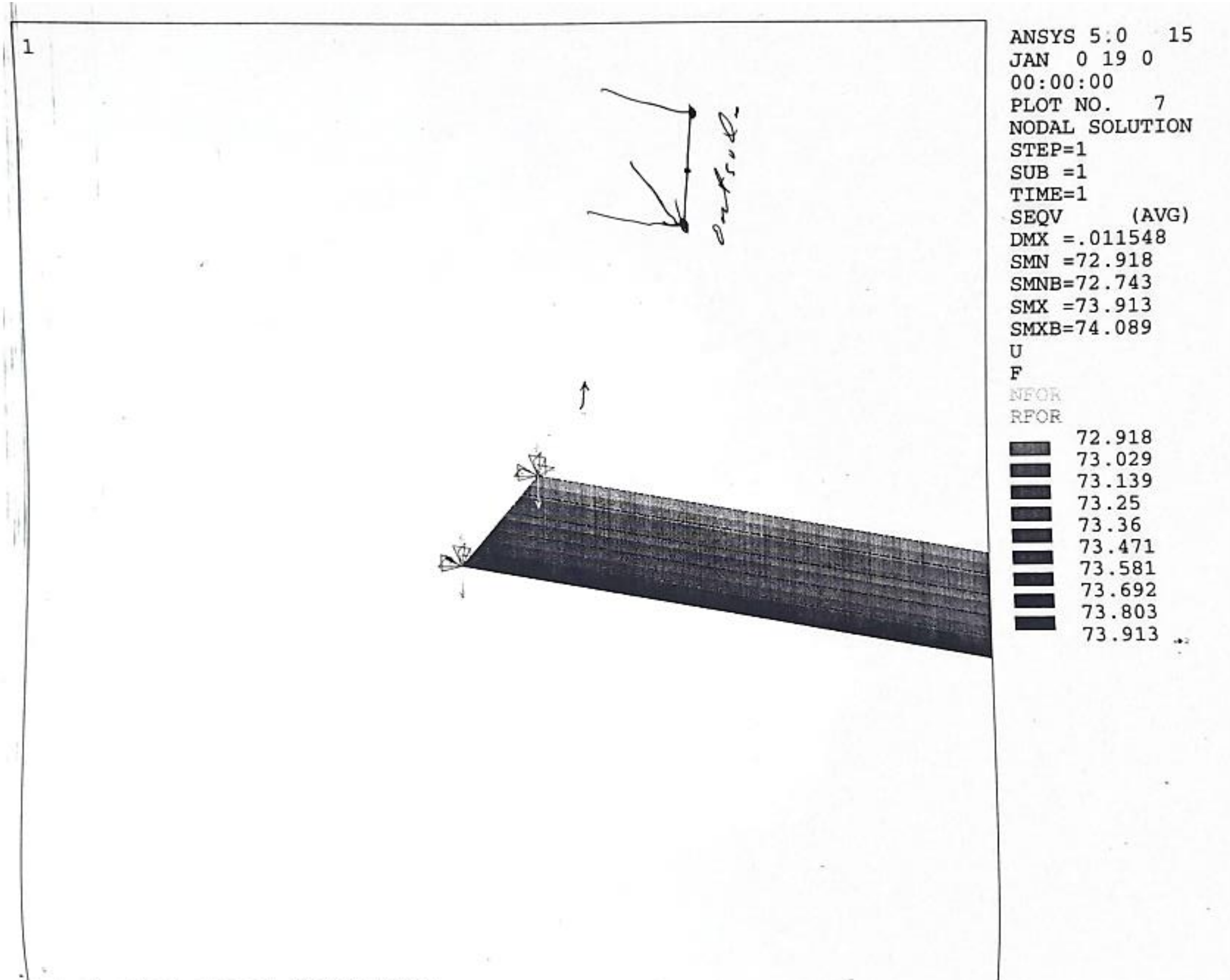
- ANALYSIS OF THE TORSIONAL SHEARING STRESS.
- TEMPERATURE PROFILE ANALYSIS ALONG RADIAL DIRECTION OF A KILN
- RADIAL AND LONGITUDINAL DEFLECTION ANALYSIS OF THE KILN'S SHELL.

FORTRAN PROGRAMS

- TEMPERATURE PROFILE ANALYSIS ALONG RADIAL DIRECTION OF A KILN.
- RADIAL DEFLECTION ANALYSIS OF THE KILN'S SHELL.
- LONGITUDINAL DEFLECTION ANALYSIS OF THE KILN'S SHELL.

ANSYS SOLUTION

ANALYSIS OF THE TORSIONAL SHEARING STRESS



Equivalent stress,distibution

```
/batch
!--------------------------------------------------------------!
!                                                              !
!                        ---------------                       !
!                      by: Gembong Baskoro                     !
!--------------------------------------------------------------!
/verify,Torque on the Shell Kiln
/com,--------------- Build the Model --------------------
/filnam,Torsi
/title,Model Problem
/unit,bin          ! British system using inch
/show,x11          ! Graphic driver

! Define parameter

RHOS=0.281         ! Mass density of Shell ( lb/in^3 )
MUSR=0.29          ! Poisson ratio of shell kiln
EXX=1.2E7          ! Young modulus of shell kiln

r=120-1.5-12       ! Inner radius of refractorie
L=1250             ! Length of Kiln
tr=12              ! thickness of refractorie
ts=1.5             ! thickness of shell
force=5.04e6/r

/prep7
/pnum,line,1
/pnum,area,1
/pnum,kpoi,1
wpave,r+tr,0
rectng,0,ts,0,L
wpave,r,0

et,1,plane25        ! Et for shell
mp,ex,1,exx         ! mp for shell

mat,1
eshape,2
esize,,5
lesize,2,,,10
amesh,1
/angle,1,-90,zs,1
/angle,1,60,xs,1
/angle,1,20,ys,1
/angle,1,-10,zs,1
/replot
finish

/solu
/pbc,all,,1
dk,1,all
dk,2,all
f,7,fz,-force
solve
/show,gem
```

```
finish
```



```
/post1
set,1
/psf,defa
/title, Equivalent stress,distibution
plnsol,s,eqv
/zoom,1,296.46,66.357,-99.723,32.209
/zoom,1,293.16,73.194,-100.46,2.5027
/zoom,1,off
lpath,17,7
/title, Graph of eqv. stresses along cross section of model
/view,all,0,0,1
/angl,all,0
plsect,s,eqv
fini
```

ANSYS SOLUTION

TEMPERATURE PROFILE ANALYSIS ALONG RADIAL DIRECTION OF A KILN

```
/batch
!
! Temperature distribution on radial direction of kiln
!
/verify,MP
/title,Temperatur distribution
/units,bin                                ! British system using inch
/show,x11                                 ! Graphic driver

!Define parameters
fci=0.0139
fco=0.0347

cb=0.015625                               ! Thermal conductivity of refraktorie
cs=0.7833                                 ! Thermal conductivity of shell kiln

Tiav=1600                                 ! Inside temperture
To=50                                     ! Outside temperature

R1=120-1.5-12                             ! Inner radius of refractorie ( in )
R2=120-1.5                                ! Inner radius of shell kiln  ( in )
R3=120                                    ! Inner radius of kiln ring ( in )

/prep7
pcirc,R2,R3,270,360
pcirc,R1,R2,270,360

et,1,plane55                              ! Define element type
mp,kxx,1,cs                               ! Material properties of shell kiln
mp,kxx,2,cb                               ! Material properties of magnesite chrome

aglue,all
aplot
eshape,2

esize,,5
lesize,7,5,15
mat,2
amesh,2

esize,,2
lesize,5,1,15
mat,1
amesh,3
finish

/solu
sfl,7,conv,fci,,Tiav
sfl,1,conv,fco,,To
solve
finish

/post1
/show,temper
set,1
/psf,defa
/edge,all,1
plnsol,temp
```

ANSYS SOLUTION

RADIAL AND LONGITUDINAL DEFLECTION ANALYSIS OF THE KILN’S SHELL

```
/batch
/verify, Deflection on the longitudinal direction
/com,
/filnam,deflek
/unit,bin                                 ! British system

! define parameter

rhos=0.281                                ! Mass density of shell ( lb/in^3 )
musr=0.29                                 ! Poisson ratio of shell kiln
exx=1.2e7                                 ! young modulus of shell

tr=1.5                                    ! thickness of shell
ri=120-tr                                 ! inner radius of shell
ro=120+tr                                 ! outer radius of shell
L=2200                                    ! Total length
L1=200
L2=1300
L3=600
L4=100

/prep7
!/pnum,line,1
!/pnum,kpoi,1
!/pnum,volu,1
/pnum,area,1

R,1,tr

csys,1
k,1
k,2,ro,-60
k,3,ro,90
k,4,ro,240

l,2,3
l,3,4
l,4,2

csys,0

k,5,0,0,-l4
k,6,0,0,-(l4+l3)
k,7,0,0,-(l4+l3+l2)
k,8,0,0,-l

l,1,5
l,5,6
l,6,7
l,7,8


/angle,1,60,ys,1
/angle,1,10,xs,1
lplot

adrag,1,2,3,,,,4
adrag,8,11,13,,,,5
adrag,14,17,19,,,,6
```

```
adrag,20,23,25,,,,7

aglue,all

nummrg,all
numcmp,all

et,1,shell63
mp,ex,1,exx
mp,nuxy,1,musr

aplot

eshape,2
lesize,1,,,30
lesize,2,,,30
lesize,3,,,6
esize,,l4/25
mat,1
amesh,1,3,1
esize,,l3/50
mat,1
amesh,4,6,1
esize,,l2/100
mat,1
amesh,7,9,1
esize,,l1/50
mat,1
amesh,10,12,1
fini

/solu
/show,hasil
/pbc,all,,1
dk,17,all
dk,14,all
dk,11,all
dk,15,uy
dk,12,uy
dk,9,uy
/psf,pres,,2
sfa,3,,pres,-250
sfa,6,,pres,-250
sfa,9,,pres,-250
sfa,12,,pres,-250
aplot
eplot
solve
save
fini

/post1
set,1
/pbc,u,,1
/psf,pres,2
/edge,all,1
pldisp,2
/title, Equivalent Stress Distribution
plnsol,s,eqv
/view,all,0,0,1
```

```
/angl,all,0
/replot
/view,all,1,0,0
/angl,1,180,ys,1
/replot
/title,Deflection
pldisp,2
/view,all,0,0,1
/angl,all,0
/replot
/view,all,1,0,0
/angl,1,180,ys,1
/replot
fini
```

FORTRAN PROGRAM

TEMPERATURE PROFILE ANALYSIS ALONG RADIAL DIRECTION OF A KILN

```
c
c Temperature distribution along r-direction of a Kiln
c Sept.,22,94 by Gembong Baskoro
c

      real T(4),Ti,To,r(4),k(4),hi,ho,c2,rr
      real sum,fac,qm,Temp
      real rmax,rmin
      integer i,a

      open(unit=10,file='temp')
      r(1)=120.-1.5-12.
      r(2)=120.-1.5
      r(3)=120
      k(1)=0.015625
      k(2)=0.7833
      Ti=1600.
      To=50.
      hi=0.0139
      ho=0.0347
      rmin=105.
      rmax=130.

      sum=0.
      do 10 i=1,2
        sum=sum+(1./k(i))*log(r(i+1)/r(i))
10    continue

      fac=(1./(r(i)*hi))+sum+(1./(r(3)*ho))
      qm=(Ti-To)/fac

      do 20 i=1,3
        if(i.eq.1) then
          T(1)=Ti-(qm/(r(i)*hi))
        else
          T(i)=T(i-1)-((qm*log(r(i)/r(i-1)))/k(i-1))
        endif
20    continue

      write(10,'(5x,a)')' Temperature on the layers '
      write(10,*)
      do 25 i=1,3
       write(10,110)i,r(i),i,T(i)
25    continue
      write(10,*)
```

```
      write(10,"(8x,21('-'))")
      write(10,'(8x,a)')'  r      Temperature'
      write(10,"(8x,21('-'))")
      write(10,*)

      do 30 rr=rmin,rmax
        if (rr.lt.r(1)) then
          a=1
          c1=(T(a)-Ti)/log(r(a)/rmin)
          c2=T(a)-c1*log(r(a))
          Temp=c1*log(rr)+c2
        elseif ( rr.ge.r(1).and.rr.le.r(3)) then
          call con(rr,T,r,c1,c2)
          Temp=c1*log(rr)+c2
        else
          a=3
          c1=(To-T(a))/log(rmax/r(a))
          c2=T(a)-c1*log(r(a))
          Temp=c1*log(rr)+c2
        endif
       write(10,100)rr,Temp
30    continue
       write(10,"(8x,21('-'))")
100   format(9x,f5.1,2x,f12.5)
110   format(1x,'r (',i1,') = ',f5.1,2x,'T (',i1,') = ',f10.5)
       close(unit=10)
       end

       subroutine con(x,T,r,c1,c2)
       real x,T(3),r(3),c1,c2
       integer i
       if (x.ge.r(1).and.x.lt.r(2)) then
          i=1
       else
          i=2
       endif

       c1=(T(i)-T(i+1))/log(r(i)/r(i+1))
       c2=T(i)-c1*log(r(i))
       return
       end
```

FORTRAN PROGRAM

RADIAL DEFLECTION ANALYSIS OF THE KILN'S SHELL

```
*-----------------------------------------------------------------------*
*                                                                       *
*                   Deflection analysis of the kiln's shell             *
*                   --------------------------------------              *
*                                        Gembong Baskoro                *
*                                                                       *
*-----------------------------------------------------------------------*

      Program deflek

      real shi,M,pi,shil,teta,mshi,tn,sec
      real R,Rx,Ry,Mx,My,inc,a,tp,del,itg
      real teta2,shi2

c open file
      open(unit=10,file="c:\mproject\mpshell")
c
c define constans
c
      pi=acos(-1.)
      shil=30.*pi/180
      tn=tan(shil)
      sec=1./cos(shil)
      Tp=2.*cos(shil)
      R=1.
      inc=2
      a=0

c
c write the heading
c
      write(10,"(1x,116('-'))")
      write(10,*)' teta   Rx    Ry     M      Mx       My',
     +'         T        Tx       Ty      del        dx      dy'
      write(10,"(1x,116('-'))")
      write(10,*)
c
c calculation procedures
c
      do 10 teta=a,180,inc
            shi=pi*teta/180.
            mshi=(1./(2.*pi))*(sec-1.-shil*tan(shil)*cos(shi))
            if (teta.gt.150) then
               M=mshi+(sin(shi+shil)/Tp)
               T=-(1./(2.*pi))*shil*tn*cos(shi)+(sin(shi+shil)/Tp)
               call inte(shi,itg)
               del=itg
            else
               M=mshi
               T=-(1./(2.*pi))*shil*tn*cos(shi)
               call intel(shi,itg)
               del=itg
            endif
c
c transform to cartesian coordinate
c
      teta2=90.-teta
      shi2=pi*teta2/180.
      Rx=R*cos(shi2)
      Ry=R*sin(shi2)
      Mx=(1.-M)*cos(shi2)
      My=(1.-M)*sin(shi2)
      Tx=(1.-T)*cos(shi2)
      Ty=(1.-T)*sin(shi2)
      dx=(1.-del)*cos(shi2)
      dy=(1.-del)*sin(shi2)
c
c write the result
c

      write(10,100)teta2,Rx,Ry,M,Mx,My,T,Tx,Ty,del,dx,dy

10    continue
100   format(1x,f5.1,11(1x,f7.4))
      close(unit=10)
      end

c subroutine for calculating integral using simpson rule
```

```
*------------------------------------------------------------------*
      subroutine inte(b,itg)
      real a,b,itg,dl,lo,fnl
      integer j
      a=0.
      m=100
      h=(b-a)/m
      dl=0.
      lo=0.
      do 10 j=1,m/2
            dl=dl+h
            lo=lo+4*fnl(dl)
            dl=dl+h
            lo=lo+2*fnl(dl)
10    continue
      itg=(h/3.)*(fnl(a)+lo-fnl(b))
      end

c function to be integrate
*------------------------------------------------------------------*
      function fnl(x)
      real fnl,pi,sec,shil
       pi=acos(-1.)
       shil=30*pi/180
       sec=1./cos(shil)
       fnl=(((1./(2.*pi))*(sec-1.-shil*tan(shil)*cos(x)))
     +  +(sin(x+shil)/(2.*cos(shil))))*sin(x)
      return
      end
*--------------------------------------------------------------------*
      subroutine intel(b,itg)
      real a,b,itg,dl,lo,fn2
      integer j
      a=0.
      m=100
      h=(b-a)/m
      dl=0.
      lo=0.
      do 10 j=1,m/2
            dl=dl+h
            lo=lo+4.*fn2(dl)
            dl=dl+h
            lo=lo+2*fn2(dl)
10    continue
      itg=(h/3.)*(fn2(a)+lo-fn2(b))
      end
c function to be integrate
*--------------------------------------------------------------------*
      function fn2(x)
      real fn2,pi,sec,shil,x
        pi=acos(-1.)
        shil=30.*pi/180
        sec=1./cos(shil)
        fn2=((1./(2.*pi))*(sec-1.-shil*tan(shil)*cos(x)))*sin(x)
        return
      end
```

FORTRAN PROGRAM

LONGITUDINAL DEFLECTION ANALYSIS OF THE KILN’S SHELL

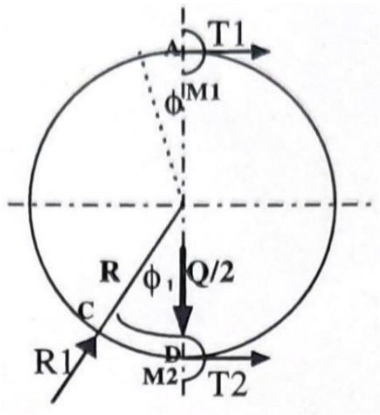
```
*-------------------------------------------------------------------------*
*                                                                         *
*                  Longitudinal Deflection of a Shell                     *
*                          Using Beam Model                               *
*                  --------------------------------                       *
*                        by: Gembong Baskoro                              *
*                                                                         *
*-------------------------------------------------------------------------*

      real q,l1,l2,l3,l4,do,di,E,I,M,Ma,Mb,Mc,la,lb,lc,ld,R1,R2,R3
      real C1,C2,C3,C4,C5,C6,C7,C8,pi,x,y,v,maxM,maxy

      open (unit=10,file='beamdat')

      pi=acos(-1.)

      q=150
      l1=200
      l2=1300
      l3=600
      l4=100

      do=243
      di=237
      E=1.2E7

      la=l1
      lb=l1+l2
      lc=l1+l2+l3
      ld=l1+l2+l3+l4

      I=(1./64.)*pi*(do**4-di**4)

      Ma=-(1./2.)*q*l1**2
      Mc=-(1./2.)*q*l4**2
      Mb=((-q/4)*(l2**3+l3**3)-Ma*l2-Mc*l3)/(2*(l2+l3))

      R1=(1./l2)*(Mb+(q/2.)*(l1+l2)**2)
      R3=(1./l3)*(Mb+(q/2.)*(l3+l4)**2)
      R2=q*ld-(R1+R3)

      C3=((q/24.)*(lb**4-la**4)+(R1/2.)*(-(2./3.)*la**3-(1./3.)*lb**3+
     +   la*lb**2))/(lb-la)
      C4=(1./24.)*(q*la**4)+(1./3.)*R1*la**3-C3*la
      C1=C3-(1./2.)*R1*la**2
      C2=(1./24.)*q*la**4-C1*la
      C5=C3+(1./2.)*R2*lb**2
      C6=(1./24.)*q*lb**4-(1./6.)*lb**3*(R1+R2)+(1./2.)*(R1*la*lb**2)+
     +   (1./2.)*R2*lb**3-C5*lb
      C7=C5+(1./2.)*R3*lc**2
      C8=(1./24.)*q*lc**4-(1./6.)*lc**3*(R1+R2+R3)+(1./2.)*lc**2*(R1*
     +   la+R2*lb+R3*lc)-C7*lc

      write(10,150)Ma,Mb,Mc
      write(10,200)R1,R2,R3
      write(10,*)
      write(10,*)'-----------------------------------------------------'
      write(10,*)'    x        Shearing      Bending     Deflection '
      write(10,*)'              force        Moment                 '
      write(10,*)'   (in)       (lb)         (lb-in)      ( in )    '
      write(10,*)'-----------------------------------------------------'

      maxM=0
      maxy=0

      do 10 x=0,ld,25
         if (x.ge.0.and.x.le.la) then
            v=-q*x
            M=-(1./2.)*q*x**2
            y=(1./(E*I))*(-(1./24.)*q*x**4+C1*x+C2)
            write(10,100)x,v,M,y
         endif
         if (x.ge.la.and.x.le.lb) then
            v=-q*x+R1
            M=-(1./2.)*q*x**2+R1*(x-la)
            y=(1./(E*I))*(-(1./24.)*q*x**4+(1./6.)*R1*x**3-(1./2.)*
     +         R1*la*x**2+C3*x+C4)
            write(10,100)x,v,M,y
         endif
         if (x.ge.lb.and.x.le.lc) then
```

```
              v=-q*x+R1+R2
              M=-(1./2.)*q*x**2+R1*(x-la)+R2*(x-lb)
              y=(1./(E*I))*(-(1./24.)*q*x**4+(1./6.)*R1*x**3-(1./2.)*
     +           R1*la*x**2+(1./6.)*R2*x**3-(1./2.)*R2*lb*x**2+C5*x+C6)
              write(10,100)x,v,M,y
            endif
            if (x.ge.lc.and.x.le.ld) then
              v=-q*x+R1+R2+R3
              M=-(1./2.)*q*x**2+R1*(x-la)+R2*(x-lb)+R3*(x-lc)
              y=(1./(E*I))*(-(1./24.)*q*x**4+(1./6.)*R1*x**3-(1./2.)*
     +           R1*la*x**2+(1./6.)*R2*x**3-(1./2.)*R2*lb*x**2+
     +           (1./6.)*R3*x**3-(1./2.)*R3*lc*x**2+C7*x+C8)
              write(10,100)x,v,M,y
            endif

        if (x.eq.la.or.x.eq.lb.or.x.eq.lc) goto 10
        if(abs(M).gt.maxM) maxM=abs(M)
        if(abs(y).gt.maxy) maxy=abs(y)

10      continue

        write(10,*)
        write(10,*)'-----------------------------------------------'
        write(10,250)maxM
        write(10,300)maxy

        close(unit=10)

100     format (1x,f7.1,3(2x,e11.5))
150     format (1x,'Ma = ',e11.5,2x,'Mb = ',e11.5,2x,'Mc = ',e11.5)
200     format (1x,'R1 = ',e11.5,2x,'R2 = ',e11.5,2x,'R3 = ',e11.5)
250     format (1x,'Maximum Bending Moment = ',e11.5,' lb-in')
300     format (1x,'Maximum Deflection     = ',e11.5,' in')
        end
```

DERIVATION FORMULA IN CHAPTER 4

A free-body sketch of the shell is shown below:

The model of Kiln's Shell



Reaction on the support:

R1 = R2 = $\frac{Q}{2\text{Cos}\phi 1}$

From the free body diagram, the bending moment M can be derived below:

between ϕ =0 and ϕ = π, is

$$M_{A-C} = M_1 - T_1.R\,(1 - \cos\phi) \qquad \text{(a)}$$

between ϕ = π - $\phi 1$ and ϕ = π, is

$$M_{C-D} = M_1 - T_1.R\,(1 - \cos\phi) - \frac{Q}{2}\,R\,\sin(\phi - \phi 1) \qquad \text{(b)}$$

Since the tangents to the shell at points A and D remain horizontal,

$$\int_A^D M d\phi = 0$$

therefore

$$\int_0^{(\pi+\phi 1)} M d\phi + \int_{(\pi-\phi 1)}^{\pi} M d\phi = 0$$

or

$$\int_0^{\pi}\{M_1 - T_1.R\,(1-\cos\phi)\}d\phi - \int_{(\pi-\phi1)}^{\pi}\frac{Q}{2}\,R\,\sin(\phi-\phi1)\,d\phi = 0$$

or

$$M_1 - T_1.R = \int_{(\pi-\phi1)}^{\pi}\frac{Q}{2}\,R\,\sin(\phi-\phi1)\,d\phi \qquad (c)$$

Since the horizontal displacement of point, A relative to point D is zero then.

$$\int_A^D M\cos\phi\,d\phi = 0$$

therefore

$$\int_0^{\pi}\{M_1 - T_1.R\,(1-\cos\phi)\}\cos\phi\,d\phi - \int_{(\pi-\phi1)}^{\pi}\frac{Q}{2}R\,\sin(\phi-\phi1)\cos\phi\,d\phi = 0$$

Thus

$$\frac{T_1.R.\pi}{2} = \int_{(\pi-\phi1)}^{\pi}\frac{Q}{2}R\,\sin(\phi-\phi1)\cos\phi\,d\phi \qquad (d)$$

Solving equations (c) and (d) simultaneously, the result is

$$M_1 = \frac{Q_1.R}{2\pi\cos\phi1}(1-\phi1.\sin\phi1.\cos\phi1+\sin^2\phi1),\; T_1 = -\frac{Q_1.R}{2\pi\cos\phi1}\sin^2\phi1 \qquad (e)$$

Finally plug equation (e) into (a), and (b) to get:

Moments

$$M_A^C = \frac{Q.R}{2\pi}(\sec\phi1 - 1 - \phi1.\tan\phi1.\cos\phi1)$$
$$M_C^D = \frac{Q.R}{2\pi}(\sec\phi1 - 1 - \phi1.\tan\phi1.\cos\phi1) + \frac{Q.R}{2\cos\phi1}\sin(\phi+\phi1)$$

in dimensionless :

$$\frac{M_A^C}{Q.R} = \frac{1}{2\pi}(\sec\phi1 - 1 - \phi1.\tan\phi1.\cos\phi1)$$
$$\frac{M_C^D}{Q.R} = \frac{1}{2\pi}(\sec\phi1 - 1 - \phi1.\tan\phi1.\cos\phi1) + \frac{Q.R}{2\cos\phi1}\sin(\phi+\phi1)$$

these results appear in chapter 4